NEAR-BRANE SU(6)-ORIGIN HIGGS IN SCHERK-SCHWARZ BREAKING OF 5-DIMENSIONAL SU(6) GUT

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The symmetry breaking of 5-dimensional SU(6) GUT is realized by Scherk-Schwarz mechanisms through trivial and pseudo non-trivial orbifold S^1/Z_2 breakings to produce dimensional deconstruction 5D SU(6)→4D SU(6). The latter also induces near-brane weakly-coupled SU(6) Baby Higgs to further break the symmetry into $\mathrm{SU}(3)_\mathrm{C}\otimes\mathrm{SU}(3)_\mathrm{H}\otimes\mathrm{U}(1)_\mathrm{C}$. The model successfully provides a scenario of the origin of (Little) Higgs from GUT scale, produces the (intermediate and light) Higgs boson with the most preferred range and establishes coupling unification and compactification scale correctly.

Keywords: Orbifold, Scherk-Schwarz breaking, Little Higgs, Standard Model

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1. Introduction

In the last decades, some efforts have been dedicated to investigate gauge theories with larger gauge symmetries inspired by the successful electroweak theory [1]. Those theories assume gauge invariances under particular symmetries larger than the SM's one, but contain $\mathrm{SU(3)_C \otimes SU(2)_L \otimes U(1)_Y}$ as a part of its subgroups at electroweak scale. One of them is the GUT model based on $\mathrm{SU(6)}$ group [2]. The model suggests that the electroweak scale physics is realized through breaking patterns $\mathrm{SU(6) \rightarrow SU(3)_C \otimes SU(3)_H \otimes U(1)_C}$, and subsequently $\mathrm{SU(3)_H \rightarrow SU(2)_L \otimes U(1)_B}$. Unfortunately the model suffers from non-existance of appropriate Higgs multiplet [3].

Following recent progress on extra dimension physics, non-Higgs mechanisms have been presented in some previous works. The so-called Scherk-Schwarz mechanisms dynamically breaks the symmetry induced by the orbifold of extra dimension [4-6]. Instead of directly breaking the symmetry, the effect of compactifying the extra dimension is considered to induce the Higgs bosons itself. This approach is known as the Higgs-gauge boson unification [7]. Recently a grand gauge-Higgs unification based on 5D $\mathrm{SU(6)}$ compactified on an orbifold S^1/Z_2 has been published with fermions in two 6-plet and one 20-plet which shows no proton decay at tree level but a little low compactification scale and heavy Higgs [20].

Little Higgs, as pseudo Nambu-Goldstone boson (PNB), can be obtained from the breaking of shift symmetry so that PNB gets the mass or global symmetry breaking has taken place [8,17-19] which in conjunction with 5D $\mathrm{SU(6)}$ the scalars can come from the fifth component of 5D gauge boson [6,7,20] and/or directly from the bulk [16].

In this paper, special conditions of Scherk-Schwarz mechanisms are utilized to resolve the problem of breaking the $\mathrm{SU(6)}$ GUT. The first trivial breaking and the second non-trivial breaking pattern are realized by compactification of orbifold S^1/Z_2 in 5-dimensional (5D) $\mathrm{SU(6)}$ in parallel at the same time, not like the trivial the (pseudo) non-trivial condition generates the scalar bosons. The unperiodic 5D scalar contains the periodic 5D scalar with extra-dimensional global symmetry for small extra dimension in the so-called near-brane area [8-23]. The global symmetry $\mathrm{SU(6)}$ comes from 5D $\mathrm{SU(6)}$ gauge symmetry based on AdS/CFT correspondence [20]. Here, in the near-brane area, the first symmetry breaking of 5D $\mathrm{SU(6)} \rightarrow$ 4D $\mathrm{SU(6)}$ is triggered by Scherk-Schwarz mechanisms and followed by trivial and pseudo non-trivial Orbifold breaking [10,11,15] to produce SU(6)-origin Little (Baby) Higgs scalar as the origin of $\mathrm{SU(6)}$ will-be-SimplestLittle and $\mathrm{SU(6)}$ Baby Higgs scalars successively. Trivial Orbifold Breaking (TOB) and Pseudo non-trivial Orbifold Breaking (POB) which perform dimensional deconstruction but still keep the symmetry intact, in principle, do produce 'exact' scalar boson, and the perturbative, incomplete series of exponential form of strongly-coupled SU(6)-origin Little (Baby) Higgses. It means that SU(6)-origin Little (Baby) Higgses decouple from and cannot exist in $\mathrm{SU(6)}$ GUT while $\mathrm{SU(6)}$ Baby Higgses are the perturbative zero mode with sextet containing triplet of Little(-like) Higgs. The $\mathrm{SU(6)}$ Baby

Higgses produce SM-like Higgses and become the topic of discussion in this paper. The second symmetry breaking of 4D SU(6)→ 4D SU(3)⊗SU(3)⊗U(1) is performed by SU(6) Little-like Higgs through orbifold-based field re-definition and the broken shift symmetry induced by the properties of VEV in lower-near-brane [15,16]. The VEVs are obtained from two Scherk-Schwarz parameters [4-7].

One can immediately predict the birth of SU(3) Little Higgses from the SU(6)-origin Little Higgses. This derivation is indeed workable and quite successful.

The paper is organized as follows, first special conditions of Scherk-Schwarz breaking, the trivial and pseudo non-trivial orbifold S^1/Z_2 breaking [15,22,24] are revealed in the next Section, then 5D model of SU(6) with 2 branes and the bulk [32,33] where gauge bosons and scalar bosons live in near-brane area ($y \sim 0$) which will provide SU(6)-origin Little Higgs, and SU(6) Baby Higgs which is basically weakly-coupled. The two have been well reconciled within the model as well as SU(6) GUT and Baby Higgs.

The pseudo non-trivial symmetry breaking to SU(3)⊗SU(3)⊗U(1) is explained in the next section. Subsequently it is shown that the emerging gauge bosons from broken 5-dimensional SU(6) could be considered as scalar boson [6,7,20] which provides the Coleman Weinberg potential for radiative symmetry breaking of 4D SU(6). Before summarizing the results, a brief discussion on the order estimations of relevant physical observables within the model is given.

2. The Scherk-Schwarz and Orbifold Breaking of SU(6)

First of all let us consider the orbifold breaking in 5D SU(6) compactified in $\mathcal{M}4 \times S^1/Z_2$. However before discussing the details, a brief review on Scherk-Schwarz mechanisms on orbifold S^1/Z_2 is given below.

2.1. *Symmetry breaking in $\mathcal{M}4 \times S^1/Z_2$ through Scherk-Schwarz mechanism*

The invariance of a theory compactified on 5-dimensional space, $\mathcal{M}4 \times S^1/Z_2$, demands $\mathcal{L}_5[\phi(x,y)] = \mathcal{L}_5[\phi(x,\tau_g(y))]$. The ordinary compactification satisfies $\phi(x,\tau_g(y)) = \phi(x,y)$ which is a special case of general Scherk-Schwarz compactification condition $\phi(x,\tau_g(y)) = T_g\phi(x,y)$ [6,10,11]. Here, $\tau_g(y)$ is the operator mapping the point y, while T_g is the twist transformation operator. Orbifold compactification has, in general, similar principles written as $\phi(x,\zeta_2(y)) = Z_2\phi(x,y)$. In case of orbifold S^1/Z_2 with one extra dimension, this has singularities at the fixed points after modding out S^1 and induces $\zeta_2(y) = -y$ which obviously satisfies the condition $\zeta_2^2 = 1$ and $Z_2^2 = 1$ with eigenvalues of ± 1 [5,6]. This means the subspace spanned by Z_2 can be generated either by $Z_2 = \pm 1$ or σ^3, and written in a diagonal bases as,

$$Z = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \pm 1 \end{pmatrix} . \tag{1}$$

Anyway, an operator T_g corresponding to certain local or global symmetry and characterizing the compactification in orbifolds which satisfies consistency condition, $T_g Z_2 T_g = Z_2$, can also be expressed as,

$$T_g = e^{2i\pi\vec{\beta}\cdot\vec{\lambda}} = e^{2i\pi\omega Q}, \tag{2}$$

where $\lambda^{a'}$ are the hermitian generators and Q is the predefined generator with a given direction in generator space, while ω and $\beta^{a'}$ are the corresponding parameters. Combining with the above consistency condition and expanding infinitesimally one immediately finds the condition [6],

$$\left\{\vec{\beta}\cdot\vec{\lambda}, Z_2\right\} = 0 \quad \text{and} \quad [T_g, Z_2] = 0. \tag{3}$$

These relations determine the broken and unbroken parts of generators under consideration. The latter also gives the singular solution $T_g = \pm 1$.

For 5D theory compactified on the S^1/Z_2 orbifold with the Scherk-Schwarz twist in Eq. (2), the twisted field obeys,

$$\phi(x, y + 2\pi R) = e^{2i\pi\omega Q}\phi(x, y), \tag{4}$$

where R is the compactification radius. Symmetry breaking is achieved if the symmetry generated by Q is broken by the 5D kinetic term and satisfies the anticommutative relation while the unbroken parts generated by Q' are determined by the second relation in Eq. (3) [6], that is

$$\{\omega Q, Z_2\} = \omega\{Q, Z_2\} = 0 \quad \text{and} \quad [Q', Z_2] = 0. \tag{5}$$

2.2. *Orbifold breaking mechanisms in 5D SU(6)*

We are now ready to apply the preceding discussion on the orbifold S^1/Z_2 to SU(6). Z_2 for SU(6) can be constructed based on 3 arrays of SU(2) type matrix along its diagonal elements as follows,

$$Z_2 = \begin{pmatrix} 1 & 0 & & & & \\ 0 & 1 & & & & \\ & & 1 & 0 & & \\ & & 0 & -1 & & \\ & & & & -1 & 0 \\ & & & & 0 & -1 \end{pmatrix}. \tag{6}$$

This form satisfies the boundary conditions of orbifold S^1/Z_2 suitable to realize the symmetry breaking SU(6)$\to$SU(3)$\otimes$SU(3)$\otimes$U(1) .

For Z_2 given in Eq. (6), the broken parts satisfying Eq. (5) are the SU(6) generators with off-diagonal elements, that is $\lambda^{\hat{a}}$ with $\hat{a} = 9, \cdots, 26$. On the contrary λ^a, with $a = 1, \cdots, 8, 27, \cdots, 35$, determines the unbroken parts. [2]

Due to orbifold singular points, the parity operator Z_2 which operates at each singular point is labeled as $Z_2^{(0)}$ for $y = 0$ and $Z_2^{(1)}$ for $y = \pi R$, $Z_2^{(0)} = Z_2^{(1)}$ as in

Eq. (6), and the following relation $U = Z_2^{(0)} Z_2^{(1)} = I_6$ holds which also provides an alternative to Eq. (6), that is, $Z_2 = U$.

Special conditions can easily be obtained from Eq. (5) which determine two special breaking patterns of orbifold S^1/Z_2 i.e. trivial orbifold breaking and the pseudo non-trivial breaking. The first is dictated by commutator of Eq. (5) setting $Z_2 = U$, then Q' comprises all generators of SU(6) which are consequently unbroken leading to dimensional deconstruction without gauge symmetry breaking [15]. The second is actually the special condition of a more general condition, with $\omega \neq 0$ and $Q \neq 0$, known as non-trivial orbifold breaking [15], coming from anti-commutator of Eq. (5) where Z_2 as in Eq. (6), $\omega \neq 0$ and Q is set to zero. This provides no broken generator eventhough field is twisted which also leads to dimensional deconstruction with intact symmetry. Accordingly both conditions give the same breaking pattern 5D-SU(6)→4D-SU(6).

The next symmetry breaking is performed by so-called SU(6) Little-like (Baby) Higgs which will be derived later with breaking pattern following the (pseudo) non-trivial one with Z_2 in Eq. (6) giving 4D-SU(6)→ 4D-SU(3)×SU(3)×U(1).

2.3. ***The 5D-model and Lagrangian***

2.3.1. *The 5D-model with 2 branes*

We adopt the 5D-model with the 4D-particles live in 2 branes and 5D-gauge bosons as well as scalar bosons live in the bulk. One brane corresponds to fixed point $y = 0$ and the other brane corresponds to another fixed point $y = \pi R$ of the orbifold S^1/Z_2 as per Fig. 1.

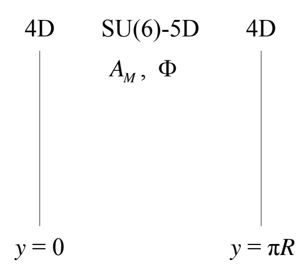

Fig. 1. 5D-model with 4D-particles live in 2-brane.

The boundary conditions consist of unitary operator U and parity operator Z_2 which have the following twisted boundary conditions

$$A_M(x, y + 2\pi R) = U A_M(x, y) U^\dagger \tag{7}$$

$$A_\mu(x, -y) = Z_2 A_\mu(x, y) Z_2^\dagger, \ A_y(x, -y) = -Z_2 A_y(x, y) Z_2^\dagger \tag{8}$$

while for fermion $\psi^f(x, -y) = \gamma_5 Z_2 \psi^f(x, y)$ where U is unitary matrix free from $y, U \in T_g$ and Z_2 is Z_2-parity transformation matrix ($Z_2^2 = I_N$). Two sets of

boundary conditions such as (U, Z_0) and (U', Z'_0) can be equivalent one to another in the sense that its physical content is the same *i.e* $(U, Z_0) \equiv (U', Z'_0)$ provided the following conditions $\partial_M U' = 0$, $\partial_M Z'_0 = 0$, $Z'^{\dagger}_0 = Z'_0$ are set [15].

Therefore Eq.(7) modifies itself as

$$A_M(x, y + 2\pi R) = A_M(x, y) \tag{9}$$

The 5D scalar boson can be obtained from Eq. (4), setting $Q = 0$ to provide Eq. (10) below under the above pseudo non-trivial breaking,

$$\widetilde{\Phi}(x, y + 2\pi R) = \widetilde{\Phi}(x, y), \tag{10}$$

where A_M is 5D SU(6) gauge bosons and $\widetilde{\Phi}$ is a single-valued and periodic function of 5D SU(6) scalar boson. From the general property of any field one can always obtain even and odd fields as follows,

$$A_M = \tfrac{1}{2}(A_M + A'_M) + \tfrac{1}{2}(A_M - A'_M) = A_M^{(+)} + A_M^{(-)}, \tag{11}$$

where the even: $A_M^{(+)}(x, y)$ and $\widetilde{\Phi}_+(x, y)$ are separated from the odds, so-called parity splitting, under the requirement of pseudo non-trivial breaking.

The scalar boson in Eq.(10) is the source of SU(6)-origin Little (Baby) Higgs boson which will produce SU(6) Baby Higgs that break the 4D-SU(6) into the above symmetry and SU(6) will-be-SimplestLittleHiggs scalar. Nevertheless one encounters the problem where Little Higgs is normally strongly-coupled near the cut-off scale of the theory while SU(6) GUT is weakly-coupled. On the other hand, the duality of orbifold breaking pattern, the trivial and the pseudo non-trivial, poses another problem due to opposite consequences of the breaking pattern. The trivial one demands for no scalar boson to exist in 4D SU(6), while the pseudo non-trivial allows scalar boson. This seemingly contradictory condition is actually what is exactly needed.

The scalar boson after the compactifying with $y \sim 0$ provides the strongly-coupled scalar which lives near the SU(6) cut-off scale so-called SU(6)-origin Little (Baby) Higgs $\widetilde{\Phi}_+^{(i)}, i = 1, 2$. Under the requirement of trivial condition this cannot exist in 4D SU(6) GUT which is automatically accomplished due to decoupling of this SU(6)-origin Little(Baby) Higgs from the much lower-energy SU(6) GUT. This fact can also be understood from coupling constant, that is, strongly-coupled scalars shall not mix with weakly-coupled SU(6) GUT. In this way the non-existant scalar in trivial breaking is accomplished. On the other hand the demand of pseudo non-trivial breaking for scalar boson can be accomplished by adjusting SU(6)-origin Little(Baby) Higgs which consist of zero mode and higher modes to the 4D SU(6) GUT requirement. If higher modes are eliminated due to the fact that 4D-content mostly resides in the zero mode term then SU(6) Baby Higgs $\widetilde{\Phi}_{+,\mathrm{P}}^{(i)}, i = 1, 2$ (weakly-coupled) is obtained by means of selecting the lowest-order expansion. Another possibility is expanding $e^{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} \sim \begin{pmatrix} e^a & e^b \\ e^c & e^d \end{pmatrix}$ to result in SU(6) will-be-SimplestLittleHiggs scalar. In this way both requirement of trivial

and pseudo non-trivial breaking are fulfilled while SU(6) Baby Higgs is also reconciled with SU(6) GUT quite well.

2.3.2. *The Origin of SU(6) scalar boson*

We can consider the 5D SU(6) scalar boson as a periodic function with the even component [16],

$$\widetilde{\Phi}_{+}(x,y) = \frac{1}{\sqrt{\pi R}}\Phi_{+}^{0}(x) + \sqrt{\frac{1}{\pi R}}\sum_{n=2}^{\infty}\Phi_{+}^{n}(x)\cos\left(\frac{ny}{R}\right) \tag{12}$$

while the odd component is,

$$\widetilde{\Phi}_{-}(x,y) = \sqrt{\frac{1}{\pi R}}\sum_{n=1}^{\infty}\Phi_{-}^{n}(x)\sin\left(\frac{ny}{R}\right). \tag{13}$$

The 5D SU(6) periodic scalar bosons must have two sets of bi-parity due to two orbifold fixed points $y = 0$ and $y = \pi R$ with combination of parity $(+, \pm)$ due to $(Z_2^{(0)}, Z_2^{(1)})$ and $(-, \pm)$ due to the same $(Z_2^{(0)}, Z_2^{(1)})$ where the first parity belongs to $y = 0$ brane.

Therefore one can write 5D-SU(6) periodic scalar bosons as the even scalar [20],

$$\widetilde{\Phi}_{+,+}(x,y) = \frac{1}{\sqrt{\pi R}}\Phi_{+,+}^{0}(x) + \sqrt{\frac{1}{\pi R}}\sum_{n=2}^{\infty}\Phi_{+,+}^{n}(x)\cos\left(\frac{ny}{R}\right), \tag{14}$$

$$\widetilde{\Phi}_{+,-}(x,y) = \frac{1}{\sqrt{\pi R}}\Phi_{+,-}^{0}(x) + \sqrt{\frac{1}{\pi R}}\sum_{n=2}^{\infty}\Phi_{+,-}^{n}(x)\cos\left(\frac{(n+\frac{1}{2})y}{R}\right) \tag{15}$$

and the odd scalars

$$\widetilde{\Phi}_{-,+}(x,y) = \sqrt{\frac{1}{\pi R}}\sum_{n=1}^{\infty}\Phi_{-,+}^{n}(x)\sin\left(\frac{(n+\frac{1}{2})y}{R}\right), \tag{16}$$

$$\widetilde{\Phi}_{-,-}(x,y) = \sqrt{\frac{1}{\pi R}}\sum_{n=1}^{\infty}\Phi_{-,-}^{n}(x)\sin\left(\frac{ny}{R}\right). \tag{17}$$

Considering the 4D-terms (parts) in Eqs. (14) and (15) which can be viewed as serial terms of exponential expression one can consider SU(6) scalar boson which lives in 4D spacetime under the cut-off scale of SU(6) theory, $\Lambda_{(6)}^{4\mathrm{D}}$, to be defined in general form as follows,

$$\widetilde{\Phi}_{+}^{(1)} = ve^{\frac{if_2}{f_1}\theta} \quad \text{and} \quad \widetilde{\Phi}_{+}^{(2)} = v'e^{-\frac{if_1}{f_2}\theta}, \tag{18}$$

where v and v' are SU(6) VEVs and θ PNB parameter which are defined later.

Lets define near-brane area as area with small y and $\alpha = \omega y/R$ where α is also relatively small then a global gauge transformation $e^{i\alpha}$ which works in periodic scalar field is obtained and produce the shift symmetry for PNB, $\frac{f_2}{f_1}\theta \to \frac{f_2}{f_1}\theta + [\alpha]$

and $\frac{f_1}{f_2}\theta \rightarrow \frac{f_1}{f_2}\theta - [\alpha]$ with $[\alpha]$: 6×6 matrix containing α, which protects the masslessness of PNB (and the SU(6) scalar boson). The shift symmetry forbids all other terms except kinetic terms in Lagrangian otherwise it is broken [19] and the global symmetry as well [6]. But SU(6) global symmetry [20] breaking can be triggered by orbifold breaking due to extra-dimensional property of α in the lower-near-brane [6,19].

The Lagrangian can be written accordingly as

$$\mathcal{L}_5^{\mathrm{SU}(6)} = D^M \Phi^\dagger D_M \Phi, \quad M = (\mu, y). \tag{19}$$

Scalar field Φ is expressed as periodic scalar field $\widetilde{\Phi}$ via the following relationship [6] where $\Phi = (\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5, \Phi_6)^T \equiv [\Phi_k], k = 1, 2, \cdots, 6$ is scalar boson in the fundamental representation of SU(6),

$$\Phi(x, y) = e^{i\omega Q_v y/R}\, \widetilde{\Phi}(x, y) = e^{iQ_v \alpha}\, \widetilde{\Phi}(x, y), \tag{20}$$

which can be obtained as solution of Eq. (4) and Q_v represents SU(6) broken generators at the direction of VEVs [6,20]. Defining $D^\mu(D_\mu)$ as 4D-covariant derivative and $D^y(D_y)$ as fifth-dimensional covariant derivative with $T^a = \lambda^a/2 (= T_a)$

$$\begin{aligned} D_\mu &= \partial_\mu - ig_5 A^a_\mu T_a,\ D^\mu = \partial^\mu + ig_5 A^\mu_a T^a \ \text{ and} \\ D_y &= \partial_y + ig_5 A^a_y T_a,\ D^y = \partial^y - ig_5 A^y_a T^a, \end{aligned} \tag{21}$$

where g_5 is the 5D coupling constant one can separate the 4D-brane from the bulk Lagrangian

$$\mathcal{L}_5^{\mathrm{SU}(6)} = \mathcal{L}_\mu^{\mathrm{brane}} + \mathcal{L}_{(\theta y),y}^{\text{near-brane}} + \mathcal{L}_y^{\mathrm{SU}(6)}, \tag{22}$$

where 4D near-brane is just in-between brane and bulk.

Thus the Lagrangians, setting $Q_v = 0$ for SU(6) upper-near-brane (thus eliminating Lorentz invariant-violating term), after Scherk-Schwarz but prior to orbifold breaking, can be expressed as follows,

$$\mathcal{L}_\mu^{\mathrm{brane}} = D^\mu \widetilde{\Phi}^\dagger D_\mu \widetilde{\Phi}, \tag{23}$$

$$\mathcal{L}_y^{\mathrm{SU}(6)} = D^y \widetilde{\Phi}^\dagger D_y \widetilde{\Phi} + ig_5 (A^a_y T_a \widetilde{\Phi} D^y \widetilde{\Phi}^\dagger - A^y_a T^a \widetilde{\Phi}^\dagger D_y \widetilde{\Phi}). \tag{24}$$

while for $\mathcal{L}_{(\theta y),y}^{\text{near-brane}}$ two cases happen and are determined by shift-symmetry-breaking parameter $(\theta\alpha)$ (or θy due to $\alpha = \frac{\omega y}{R}$) as follows: In the upper-near-brane where shift symmetry is intact and (θy)-term is negligible so that θ is small (and also for the reason which will be clear after Eq.(28) where gauge-scalar unification [15] is applied) one finds, based on Eq.(18), as follows

$$(y \sim 0, (\theta y) \to 0),\ D^\mu \widetilde{\Phi}^\dagger = D_\mu \widetilde{\Phi} = 0 \to \text{ due to } \partial^\mu \theta \text{ (or } \partial_\mu \theta) \to 0,$$

while in the lower-near-brane where shift symmetry is broken (θy)-term and also θ are significant one finds the following,

$$(y \sim 0, (\theta y) \to \mu \neq 0),\ D^\mu \widetilde{\Phi}^\dagger \neq D_\mu \widetilde{\Phi} = \text{significant value of } \partial^\mu \theta (\partial_\mu \theta).$$

Adjusting notation for both lower- and upper-near-brane, by means of $\mathcal{L}^{\text{near-brane}}_{(\theta y),y} = \mathcal{L}^{\text{near-brane}}_{\mu}$ and $\mathcal{L}^{\text{near-brane}}_{(\theta y),y} = \mathcal{L}^{\text{near-brane}}_{y}$, one finally obtains, for upper-near-brane:

$$\mathcal{L}^{\text{near-brane}}_{y} = \delta(y)\left\{\tfrac{1}{2}g_5^2\widetilde{\Phi}^\dagger A^y_a A^a_y \widetilde{\Phi}\right\}, \tag{25}$$

(and for lower-near-brane: for the reason which will be clear after Eq.(28) and Subsection 3.3 as,

$$\mathcal{L}^{\text{near-brane}}_{\mu} = V^{(6)}_{\mu},$$

where $V^{(6)}_{\mu}$ will turn out to be $V^{(6)}_{\mu\text{P}}$ in Eq.(50)).

Eq. (23) reflects the condition in the brane $(y = 0, \mu)$ while Eq. (24) of the far-distant out-of-brane condition $(y > 0, \mu)$.

3. Near-Brane 4D Scalar Bosons

3.1. *Upper-near-brane extra-strongly-coupled scalar boson (SU(6)-origin Little(Baby) Higgs scalar boson)*

In the brane $y = 0$ and near-brane $y \sim 0$ the even scalar bosons in Eqs. (14) and (15) become as

$$\widetilde{\Phi}^{(1)}_{+}(x) = \widetilde{\Phi}_{+,+}(x,y)\mid_{y=0\,\text{or}\sim 0}, \quad \widetilde{\Phi}^{(2)}_{+}(x) = \widetilde{\Phi}_{+,-}(x,y)\mid_{y=0\,\text{or}\sim 0}. \tag{26}$$

For the upper-near-brane area Neumann boundary condition dictates the following $D^y\widetilde{\Phi}^\dagger = D_y\widetilde{\Phi} = 0$ and based on property of extra-dimensional dominance $D^\mu\widetilde{\Phi}^\dagger = D_\mu\widetilde{\Phi} = 0$ make Eqs. (23) and (24) zero. This shows that Eq. (25) is really the only upper-near-brane equation with $\delta(y) = 1$ for $y \sim 0$ and under both trivial and pseudo non-trivial breaking condition, $Q_v = 0$, the final value has been obtained,

$$\mathcal{L}^{\text{near-brane}}_{y} = \tfrac{1}{2}g_5^2\left(\widetilde{\Phi}^{(i)\dagger}A^y_a\right)\left(A^a_y\widetilde{\Phi}^{(i)}\right), \tag{27}$$

where now $\widetilde{\Phi}^{(i)} = \widetilde{\Phi}^{(i)}_{+}(x)$ and $\widetilde{\Phi}^{(i)\dagger} = \widetilde{\Phi}^{(i)\dagger}_{+}(x)$, with $i = 1, 2$.

In this upper-near-brane bulk $(y-\text{area})$, under the provision of pseudo non-trivial orbifold breaking where $A^y_{\hat{a}}T^{\hat{a}}$ and $A^{\hat{a}}_yT_{\hat{a}}$ produce upper-near-brane scalar due to gauge-scalar unification [15], one has the subsets (sextet out of 2×9 broken $A^y_{\hat{a}}$ and $A^{\hat{a}}_y$) from Eq. (8)

$$A^y_{\hat{a}}T^{\hat{a}} \supset \widetilde{\Phi}^{(j)}, \quad A^{\hat{a}}_yT_{\hat{a}} \supset \widetilde{\Phi}^{(j)\dagger} \tag{28}$$

where $\widetilde{\Phi}^{(j)}(\text{or}\,\widetilde{\Phi}^{(j)\dagger})$ is diagonal 3×3 sub-matrix component of 6×6 matrix of $A^y_{\hat{a}}T^{\hat{a}}$ (or $A^{\hat{a}}_yT_{\hat{a}}$) and $j = 1, 2$ due to hermitian conjugacy and the following $D^\mu\widetilde{\Phi}^\dagger = D^\mu A^{\hat{a}}_yT_{\hat{a}} = 0$ and $D_\mu\widetilde{\Phi} = D_\mu A^y_{\hat{a}}T^{\hat{a}} = 0$. On the other side in the lower-near-brane $D^\mu\widetilde{\Phi}^\dagger \neq D_\mu\widetilde{\Phi} \neq 0$ due to significant $\partial^\mu\theta(\partial_\mu\theta)$ because of dominant 4D-property. The only term of Eq. (27) provides the quartic term with index (i) to label the original scalar boson in Eq. (27) which can be rewritten, using $\widetilde{\Phi}^{(j)}$ as diagonal component of $A^y_{\hat{a}}T^{\hat{a}}$, as

$$V^{(6)}_y = \lambda^{(6)}_y(\widetilde{\Phi}^{(i)\dagger}\widetilde{\Phi}^{(j)})(\widetilde{\Phi}^{(j)\dagger}\widetilde{\Phi}^{(i)}) \tag{29}$$

with $\lambda_y^{(6)} = g_5^2$. If one takes $g_5 \sim \mathcal{O}(1)$ then $\lambda_y^{(6)} \sim \mathcal{O}(1)$ to $\mathcal{O}(10)$ in Eq.(29) which reflects a very strong quartic coupling of typical Coleman-Weinberg potential. Since $\widetilde{\Phi}^{(j)}(\widetilde{\Phi}^{(j)\dagger})$ is arbitrarily taken from 9 broken $A^y_{\hat{a}}T^{\hat{a}}$ (and 9 broken $A^{\hat{a}}_y T_{\hat{a}}$) it is justified to take $i \neq j$ in Eq. (29).

In the near-brane with extra-strongly-interacting SU(6) scalars, the potential in Eq. (29) is zero as required by shift symmetry on Pseudo Nambu-Goldstone Boson (PNB), θ. Therefore one can call $\widetilde{\Phi}^{(1)}_+(x)$ and $\widetilde{\Phi}^{(2)}_+(x)$ as the SU(6)-origin Little (Baby) Higgs which must be defined by, making use the most basic original parameters, radius of compactification R, two Higgs doublets h and h' and two Scherk-Schwarz parameters ω_1 and ω_2 following $i \neq j = 1, 2$. Here can one also defines two VEVs, v and v' in accordance to two Scherk-Schwarz parameters at near-brane $y \sim 0$ as,

$$v = \begin{pmatrix} 0 \\ 0 \\ \frac{\omega_1}{R} \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{\pi R}} \begin{pmatrix} 0 \\ 0 \\ f_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, v' = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{\omega_2}{R} \end{pmatrix} = \frac{1}{\sqrt{\pi R}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ f_2 \end{pmatrix}, \tag{30}$$

$$f_1 = \frac{\omega_1 \sqrt{\pi}}{\sqrt{R}}, f_2 = \frac{\omega_2 \sqrt{\pi}}{\sqrt{R}}. \tag{31}$$

The parameterization of SU(6)-origin Little (Baby) Higgs is governed by the number of scalar doublets which are allowed to be put in 6×6 matrix. Thus it depends on the number of generated PNBs through the condition $a'_{jk}\widetilde{\Phi}_k \neq 0$ with $a' = 1, \cdots, 35$ and $\langle \widetilde{\Phi}^{(1)} \rangle = v$, $\langle \widetilde{\Phi}^{(2)} \rangle = v'$. These determine the total number of PNBs to be 22. However, the simplest Little Higgs at SU(3)×SU(3) requires only 10 PNBs while SU(2)×U(1) produces 4 PNBs to become U(1) . Therefore one may yet have free 8 scalar bosons which could create 4 scalar doublets to be assigned as the SU(6)-origin Little (Baby) Higgs as follows,

$$\theta = \frac{1}{f} \begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} (0)_{2\times2} & (h)_{2\times1} \\ (h'^{\dagger})_{1\times2} & 0 \end{pmatrix} \\ \begin{pmatrix} (0)_{2\times2} & (h')_{2\times1} \\ (h^{\dagger})_{1\times2} & 0 \end{pmatrix} & (0)_{3\times3} \end{pmatrix} \tag{32}$$

where $f^2 = f_1^2 + f_2^2$. The scalar doublets h and h' are the would-be SM Higgs as will be clarified later.

Therefore Eq. (18) can be reexpressed in complete forms, for $\widetilde{\Phi}^{(1)}_+$ and $\widetilde{\Phi}^{(2)}_+$ successively, as

$$\frac{1}{\sqrt{\pi R}} e^{\frac{if_2}{f_1 f}\begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} 0&0&h\\ 0&0&h'^{\dagger}\\ &&0\end{pmatrix} \\ \begin{pmatrix} 0&0&h'\\ 0&0&\\ h^{\dagger}&0&\end{pmatrix} & (0)_{3\times3}\end{pmatrix}} \begin{pmatrix}0\\0\\f_1\\0\\0\\0\end{pmatrix} \text{ and } \frac{1}{\sqrt{\pi R}} e^{-\frac{if_1}{f_2 f}\begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} 0&0&h\\ 0&0&h'^{\dagger}\\ &&0\end{pmatrix} \\ \begin{pmatrix} 0&0&h'\\ 0&0&\\ h^{\dagger}&0&\end{pmatrix} & (0)_{3\times3}\end{pmatrix}} \begin{pmatrix}0\\0\\0\\0\\0\\f_2\end{pmatrix}. \tag{33}$$

Under the requirement of shift symmetry SU(6)-origin Little (Baby) Higgs scalar remains massless due to masslessness of PNB. The breaking of shift symmetry leads to the massiveness of PNB.

From Eq.(18) and Eq. (33) it is clear $\widetilde{\Phi}_+^{(i)\dagger}\widetilde{\Phi}^{(j)} = 0, i \neq j = 1,2$ and quartic potential in Eq.(29) becomes zero. This shows the SU(6)-origin Little (Baby) Higgses are still massless and interact one to another by means of quantum interaction and is called Extra-Strongly-Coupled (ESC). Here minimum potential is always zero so that the introduction of VEVs does not bring about SU(6) nor shift symmetry breaking. Reviewing again the higher terms of $e^{i\left(\frac{f_2}{f_1}\theta+\alpha Q_v\right)}$ or $e^{-i\left(\frac{f_1}{f_2}\theta-\alpha Q_v\right)}$ and making use Eq. (32) one finds mixed terms-$\alpha\theta Q_v$ (with factor $\frac{\alpha}{f}$) while neglecting $\alpha^2 Q_v^2, \theta^2, (\theta+\alpha Q_v)^3$ etc, which gives two interval of values i.e $\frac{\alpha}{f}$ neglectable or $\frac{\alpha}{f}$ significant. The first happens in upper (farther) part of near-brane while the second shows up in the lower (nearer) part of near-brane i.e shift symmetry is broken in the lower-near-brane.

If we expand $\widetilde{\Phi}_+^{(1)}$ and $\widetilde{\Phi}_+^{(2)}$ above the expressions in Eq.(14) and Eq. (15) are obtained immediately for $y = 0 (y \sim 0)$ with zero mode and higher modes defined as follows,

$$\text{for } \tilde{\Phi}_+^1 : \tilde{\Phi}_{+,+}^0(x) = \left\{1 + \frac{if_2}{f_1 f}\begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} 0&0&h\\ 0&0&h'^{\dagger}\\ &&0\end{pmatrix} \\ \begin{pmatrix} 0&0&h'\\ 0&0&\\ h^{\dagger}&0&\end{pmatrix} & (0)_{3\times3}\end{pmatrix}\right\}\begin{pmatrix}0\\0\\f_1\\0\\0\\0\end{pmatrix}, \tag{34}$$

$$\text{for } \tilde{\Phi}_+^2 : \tilde{\Phi}_{+,-}^0(x) = \left\{1 - \frac{if_1}{f_2 f}\begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} 0&0&h\\ 0&0&h'^{\dagger}\\ &&0\end{pmatrix} \\ \begin{pmatrix} 0&0&h'\\ 0&0&\\ h^{\dagger}&0&\end{pmatrix} & (0)_{3\times3}\end{pmatrix}\right\}\begin{pmatrix}0\\0\\0\\0\\0\\f_2\end{pmatrix}, \tag{35}$$

and for both:

$$\widetilde{\Phi}_{+,+}^n(x)\left[\widetilde{\Phi}_{+,-}^n(x)\right] = \frac{1}{n!}\left\{\frac{if_2}{f_1 f}\left[-\frac{if_1}{f_2 f}\right]\begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} 0&0&h\\ 0&0&h'^{\dagger}\\ &&0\end{pmatrix} \\ \begin{pmatrix} 0&0&h'\\ 0&0&\\ h^{\dagger}&0&\end{pmatrix} & (0)_{3\times3}\end{pmatrix}\right\}^n \begin{pmatrix}0\\0\\f_1\\0\\0\\0\end{pmatrix}\left[\begin{pmatrix}0\\0\\0\\0\\0\\f_2\end{pmatrix}\right], \tag{36}$$

$$n = 2, 3, \ldots, \infty.$$

These give perturbative expressions of SU(6)-origin Little (Baby) Higgses from which the so-called SU(6) Baby Higgses are defined in this paper as lowest-order

of the expansion i.e. the zero mode. Consequently one must establish a cut-off scale for perturbative approach $\Lambda^{\rm NP}_{(6)}$ above which only the ESC SU(6)-origin Little(Baby) Higgs theory takes control i.e. $\Lambda^{\rm NP}_{(6)} <$ ESC regime (upper part) $< \Lambda^{\rm 4D}_{(6)}$.

3.2. *Lower-near-brane strongly-coupled scalar boson (SU(6) will-be-SimplestLittleHiggs scalar boson)*

Now lets discuss the mechanisms to generate Simplest Little Higgs from SU(6)-origin Little(Baby) Higgs scalar in Eq. (33), assuming $\mathcal{O}(f)\alpha\mathcal{O}(f_i)$ yields $f_2/f_1 f$ and $f_1/f_2 f \ll 1$. Then can one perform the following expansion

$$e^{\frac{if_2}{f_1 f}\left[-\frac{if_1}{f_2 f}\right]\begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} 0 & 0 & h \\ 0 & 0 & \\ h'^\dagger & & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & h' \\ 0 & 0 & \\ h^\dagger & & 0 \end{pmatrix} & (0)_{3\times3} \end{pmatrix}} \sim \begin{pmatrix} 1 & e^{\frac{if_2}{f_1 f}\left[-\frac{if_1}{f_2 f}\right]\begin{pmatrix} 0 & 0 & h \\ 0 & 0 & \\ h'^\dagger & & 0 \end{pmatrix}} \\ e^{\frac{if_2}{f_1 f}\left[-\frac{if_1}{f_2 f}\right]\begin{pmatrix} 0 & 0 & h' \\ 0 & 0 & \\ h^\dagger & & 0 \end{pmatrix}} & 1 \end{pmatrix} \tag{37}$$

to obtain as follows,

$$\widetilde{\Phi}^{(1)'}_{+} = \begin{pmatrix} \phi^{(1)}_0 \\ \phi^{(1)} \end{pmatrix} \quad \text{and} \quad \widetilde{\Phi}^{(2)'}_{+} = \begin{pmatrix} \phi^{(2)} \\ \phi^{(2)}_0 \end{pmatrix}, \tag{38}$$

where $\widetilde{\Phi}^{(i)'}_{+}, i = 1,2$ is the SU(6) will-be-SimplestLittleHiggs scalar and lives below the scale $\Lambda^{\rm NP}_{(6)}$,

$$\phi^{(1)}_0 = \frac{1}{\sqrt{\pi R}}\begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix}, \quad \phi^{(1)} = \frac{1}{\sqrt{\pi R}} e^{\frac{if_2}{f_1 f}\begin{pmatrix} 0 & 0 & h' \\ 0 & 0 & \\ h^\dagger & & 0 \end{pmatrix}} \begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix}, \tag{39}$$

$$\phi^{(2)}_0 = \frac{1}{\sqrt{\pi R}}\begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix}, \quad \phi^{(2)} = \frac{1}{\sqrt{\pi R}} e^{-\frac{if_1}{f_2 f}\begin{pmatrix} 0 & 0 & h \\ 0 & 0 & \\ h'^\dagger & & 0 \end{pmatrix}} \begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix}. \tag{40}$$

This must happen after f reaches significantly lower value than $\Lambda^{\rm NP}_{(6)}$ in the lower-near-brane so that $\frac{\alpha}{f}$ of $\alpha\theta$-term becomes significant and shift symmetry is broken. Massive pseudo Nambu-Goldstone boson (PNB) is absorbed by scalar to become VEV, $\phi^{(1)}_0$ and $\phi^{(2)}_0$. Defining the scale $f'_i = \frac{1}{\sqrt{\pi R}} f, H(H') = \frac{1}{\sqrt{\pi R}} h(h')$ finally one can rewrite the SU(3) Little Higgs as,

$$\phi^{(1)} = e^{\frac{if'_2}{f'_1 f'}\begin{pmatrix} 0 & 0 & H' \\ 0 & 0 & \\ H^\dagger & & 0 \end{pmatrix}} \begin{pmatrix} 0 \\ 0 \\ f'_1 \end{pmatrix}, \quad \phi^{(2)} = e^{-\frac{if'_1}{f'_2 f'}\begin{pmatrix} 0 & 0 & H \\ 0 & 0 & \\ H'^\dagger & & 0 \end{pmatrix}} \begin{pmatrix} 0 \\ 0 \\ f'_2 \end{pmatrix}. \tag{41}$$

In case of $H = H'$, Eq. (41) become basically the Simplest Little Higgs on SU(3)×SU(3) as expected [8].

3.3. *Lower-near-brane Weakly-coupled scalar boson (SU(6) Baby Higgs)*

This is represented by SU(6) Baby Higgses which are defined by zero mode approximation where perturbative approach has been taken up to lowest order. This scalar lives below energy scale $\Lambda^{\rm NP}_{(6)}$. SU(6) Baby Higgses can be written as (P : perturbative),

$$\widetilde{\Phi}^{(1)}_{+,\rm P}(x) = v\left(1+\frac{if_2}{f_1}\theta(x)\right), \quad \widetilde{\Phi}^{(2)}_{+,\rm P}(x) = v'\left(1-\frac{if_1}{f_2}\theta(x)\right), \tag{42}$$

or, in complete forms as follows

$$\widetilde{\Phi}^{(1)}_{+,\rm P}(x) = \frac{1}{\sqrt{\pi R}}\left\{1+\frac{if_2}{f_1 f}\begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} 0 & 0 & h \\ 0 & 0 & \\ h'^{\dagger} & & 0\end{pmatrix} \\ \begin{pmatrix} 0 & 0 & h' \\ 0 & 0 & \\ h^{\dagger} & & 0\end{pmatrix} & (0)_{3\times3}\end{pmatrix}\right\}\begin{pmatrix}0\\0\\f_1\\0\\0\\0\end{pmatrix}, \tag{43}$$

$$\widetilde{\Phi}^{(2)}_{+,\rm P}(x) = \frac{1}{\sqrt{\pi R}}\left\{1-\frac{if_1}{f_2 f}\begin{pmatrix} (0)_{3\times3} & \begin{pmatrix} 0 & 0 & h \\ 0 & 0 & \\ h'^{\dagger} & & 0\end{pmatrix} \\ \begin{pmatrix} 0 & 0 & h' \\ 0 & 0 & \\ h^{\dagger} & & 0\end{pmatrix} & (0)_{3\times3}\end{pmatrix}\right\}\begin{pmatrix}0\\0\\0\\0\\0\\f_2\end{pmatrix}, \tag{44}$$

or, making use VEVs in Eq. (30),

$$\widetilde{\Phi}^{(1)}_{+,\rm P}(x) = v+\frac{1}{\sqrt{\pi R}}\begin{pmatrix}0\\0\\0\\ \frac{if_2}{f_1 f}\begin{pmatrix} 0 & 0 & h' \\ 0 & 0 & \\ h^{\dagger} & & 0\end{pmatrix}\begin{pmatrix}0\\0\\f_1\end{pmatrix}\end{pmatrix}, \widetilde{\Phi}^{(2)}_{+,\rm P}(x)= v'-\frac{1}{\sqrt{\pi R}}\begin{pmatrix}\frac{if_1}{f_2 f}\begin{pmatrix} 0 & 0 & h \\ 0 & 0 & \\ h'^{\dagger} & & 0\end{pmatrix}\begin{pmatrix}0\\0\\f_2\end{pmatrix}\\0\\0\\0\end{pmatrix}. \tag{45}$$

Eq. (45) brings us immediately to the orbifold-based field redefinition as follows,

$$\widetilde{\Phi}^{(1)'}_{+,\rm P}(x) = \widetilde{\Phi}^{(1)}_{+,\rm P}(x) - v + v', \quad \widetilde{\Phi}^{(2)'}_{+,\rm P}(x)= \widetilde{\Phi}^{(2)}_{+,\rm P}(x) - v' + v. \tag{46}$$

The new SU(6) Baby Higgses are surprisingly split into triplets of SU(3) Little-like Higgses in accordance to (x is not written for simplicity),

$$\widetilde{\Phi}^{(1)'}_{+,\rm P}(x) = \begin{pmatrix} 0_{3\times1} \\ \phi^{(1)}_P\end{pmatrix}, \quad \widetilde{\Phi}^{(2)'}_{+,\rm P}(x)= \begin{pmatrix} \phi^{(2)}_P \\ 0_{3\times1}\end{pmatrix}, \tag{47}$$

where SU(3) Little-like Higgses triplets are defined and obtained as

$$\phi^{(1)}_{\rm P} = \frac{1}{\sqrt{\pi R}}\left[\left\{\left(1+\frac{\Delta f}{f_1}\right)+\frac{if_2}{f_1 f}\begin{pmatrix} 0 & 0 & h' \\ 0 & 0 & \\ h^{\dagger} & & 0\end{pmatrix}\right\}\begin{pmatrix}0\\0\\f_1\end{pmatrix}\right], \tag{48}$$

$$\phi^{(2)}_{\rm P} = \frac{1}{\sqrt{\pi R}}\left[\left\{\left(1-\frac{\Delta f}{f_2}\right)-\frac{if_1}{f_2 f}\begin{pmatrix} 0 & 0 & h \\ 0 & 0 & \\ h'^{\dagger} & & 0\end{pmatrix}\right\}\begin{pmatrix}0\\0\\f_2\end{pmatrix}\right], \tag{49}$$

where $\Delta f = f_2 - f_1$.

Eq. (42) shows, under global gauge transformation $e^{i\alpha Q_v}$ (α small), the conservation of shift symmetry due to negligible $\alpha Q_v\theta$, that is $\frac{f_2}{f_1}\alpha Q_v\theta \ll 1, \frac{f_1}{f_2}\alpha Q_v\theta \ll 1$ in the following terms $\left\{\left(1+\frac{if_2}{f_1}\theta\right)+i\alpha Q_v-\frac{f_2}{f_1}\alpha Q_v\theta\right\}$ and

$\left\{\left(1-\frac{if_1}{f_2}\theta\right)+i\alpha Q_v+\frac{f_1}{f_2}\alpha Q_v\theta\right\}$. SU(6) Baby Higgs remains massless until field redefinition is performed which lowers down SU(6) to SU(3) scale. Shift symmetry is broken via Eqs. (47)-(49) which make $\alpha Q_v\theta$-term becoming significant with respect to $(f_2/f_1\theta+\alpha Q_v)$ and $(f_1/f_2\theta-\alpha Q_v)$, so PNB gets mass when global symmetry is broken in the lower-near-brane.

The potential of SU(6) Baby Higgses follows from Eq. (29) by replacing $\lambda_y^{(6)}\to\lambda_{\mu\mathrm{P}}^{(6)}$,

$$V_{\mu\mathrm{P}}^{(6)}=\delta_{ij}\lambda_{\mu\mathrm{P}}^{(6)}\widetilde{\Phi}_{+,\mathrm{P}}^{(i)'\dagger}\widetilde{\Phi}_{+,\mathrm{P}}^{(j)'}\widetilde{\Phi}_{+,\mathrm{P}}^{(j)'\dagger}\widetilde{\Phi}_{+,\mathrm{P}}^{(i)'},\tag{50}$$

where it is clear from Eqs. (47), (48) and (49) that $\widetilde{\Phi}_{+,\mathrm{P}}^{(1)'}\sim\left(\begin{smallmatrix}0_{3\times1}\\ \left(\begin{smallmatrix}0\\0\\f_1/\sqrt{\pi R}\end{smallmatrix}\right)\end{smallmatrix}\right)$ and $\widetilde{\Phi}_{+,\mathrm{P}}^{(2)'}\sim\left(\begin{smallmatrix}\left(\begin{smallmatrix}0\\0\\f_2/\sqrt{\pi R}\end{smallmatrix}\right)\\ 0_{3\times1}\end{smallmatrix}\right)$ for i and $j=1,2$. One concludes that $\widetilde{\Phi}_{+,\mathrm{P}}^{(i)'}=\widetilde{\Phi}_{+,\mathrm{P}}^{(j)'}$ for $i=j$ and $\widetilde{\Phi}_{+,\mathrm{P}}^{(i)'}\sim\widetilde{\Phi}_{+,\mathrm{P}}^{(j)'}$ for $i\neq j$. From Eq. (47) one finds $\widetilde{\Phi}_{+,\mathrm{P}}^{(i)'\dagger}\widetilde{\Phi}_{+,\mathrm{P}}^{(j)'}=0$ for $i\neq j$. Therefore Eq. (50) is rewritten as

$$V_{\mu\mathrm{P}}^{(6)}=\lambda_{\mu\mathrm{P}}^{(6)}\left\{\left(\widetilde{\Phi}_{+,\mathrm{P}}^{(1)'\dagger}\widetilde{\Phi}_{+,\mathrm{P}}^{(1)'}\right)^2+\left(\widetilde{\Phi}_{+,\mathrm{P}}^{(2)'\dagger}\widetilde{\Phi}_{+,\mathrm{P}}^{(2)'}\right)^2\right\},\tag{51}$$

and one finally arrives at the following identities,

$$\widetilde{\Phi}_{+,\mathrm{P}}^{(1)'\dagger}\widetilde{\Phi}_{+,\mathrm{P}}^{(1)'}=\phi_{\mathrm{P}}^{(1)\dagger}\phi_{\mathrm{P}}^{(1)}\ \text{ and }\ \widetilde{\Phi}_{+,\mathrm{P}}^{(2)'\dagger}\widetilde{\Phi}_{+,\mathrm{P}}^{(2)'}=\phi_{\mathrm{P}}^{(2)\dagger}\phi_{\mathrm{P}}^{(2)}.\tag{52}$$

After making use of Eqs. (48) and (49) and applying the following approach

$$\left(\frac{\Delta f'}{f_i'}\right)^2\sim\left(\frac{\Delta f'}{f_i'f'}\right)\sim0,\tag{53}$$

where $f_i'=\frac{f_i}{\sqrt{\pi R}}, i=1,2., f'^2=f_1'^2+f_2'^2, \Delta f'=f_2'-f_1'$ and $H(H')=\frac{h}{\sqrt{\pi R}}\left(\frac{h'}{\sqrt{\pi R}}\right)$ a new field shall be defined as $H''=H'-H$ which has the order of SM VEV i.e. $\mathcal{O}(\langle H''\rangle)\sim\mathcal{O}(v'')\sim\mathcal{O}(100\text{ GeV})$ which will be clarified later [37]. These provide the following,

$$\phi_{\mathrm{P}}^{(1)\dagger}\phi_{\mathrm{P}}^{(1)}=f_1'^2+2\Delta f'f_1'+\frac{if_2'^2}{f'}\left(\begin{smallmatrix}0\ 0 & H''\\ 0\ 0 & \\ -H''^{\dagger} & 0\end{smallmatrix}\right)+\frac{f_2'^2}{f'^2}\left(\begin{smallmatrix}HH^{\dagger} & 0\\ 0 & (H'^{\dagger}H')\end{smallmatrix}\right),\tag{54}$$

$$\phi_{\mathrm{P}}^{(2)\dagger}\phi_{\mathrm{P}}^{(2)}=f_2'^2-2\Delta f'f_2'+\frac{if_1'^2}{f'}\left(\begin{smallmatrix}0\ 0 & H''\\ 0\ 0 & \\ -H''^{\dagger} & 0\end{smallmatrix}\right)+\frac{f_1'^2}{f'^2}\left(\begin{smallmatrix}H'H'^{\dagger} & 0\\ 0 & (H^{\dagger}H)\end{smallmatrix}\right).\tag{55}$$

Besides the existing Baby Higgses field H and H' a new field H'' has emerged which will be clear later on as the SM-like Higgs. Having substituted Eqs. (54), (55) into (52) and further into (51), taking the mass terms and quartic terms, neglecting the constant field, $V_{\mu\mathrm{P}}^{(6)}$ now can be decomposed into 3 parts i.e potential of H'', H' and H and rewritten as

$$V_{\mu\mathrm{P}}^{(6)}=V_{H''}^{(6)}+V_{H'}^{(6)}+V_{H}^{(6)}\tag{56}$$

where

$$V^{(6)}_{H''} = \lambda^{(6)}_{\mu\mathrm{P}} \frac{f_1'^4 + f_2'^4}{f'^2} H''^{\dagger} H'', \tag{57}$$

$$V^{(6)}_{H'} = \lambda^{(6)}_{\mu\mathrm{P}} \left(\frac{f_1'^2 f_2'^2}{f'^2} + \frac{(2\Delta f' f_1') f_2'^2}{f'^2} \right) H'^{\dagger} H' + \lambda^{(6)}_{\mu\mathrm{P}} \frac{f_2'^4}{f'^4} (H'^{\dagger} H')^2, \tag{58}$$

$$V^{(6)}_{H} = \lambda^{(6)}_{\mu\mathrm{P}} \left(\frac{f_1'^2 f_2'^2}{f'^2} - \frac{f_1'^2 (2\Delta f' f_2')}{f'^2} \right) H^{\dagger} H + \lambda^{(6)}_{\mu\mathrm{P}} \frac{f_1'^4}{f'^4} (H^{\dagger} H)^2. \tag{59}$$

All Eqs.(57), (58) and (59) have the mass terms now proving the broken shift symmetry. These show the important property of weakly-coupled potential and strongly indicate that $\lambda^{(6)}_{\mu\mathrm{P}}$ is relatively small compared to $\lambda^{(6)}_{y}$. This also justifies that in weakly-coupled regime the interaction takes place at tree level which can produce mass.

If one assumes that $\Delta f' \ll f_1' \sim f_2' \sim f'$ in SU(6) scale then $V^{(6)}_{H'} \sim V^{(6)}_{H}$. In order to simplify further let's also assume $H \sim H'$ then one finds,

$$V^{(6)}_{H'} + V^{(6)}_{H} = \lambda^{(6)}_{\mu\mathrm{P}} \frac{2 f_1'^2 f_2'^2}{f'^2} H^{\dagger} H + \lambda^{(6)}_{\mu\mathrm{P}} \frac{f_1'^4 + f_2'^4}{f'^4} (H^{\dagger} H)^2 \ \sim 2 V^{(6)}_{H} \tag{60}$$

in which the radiative mass term of Little-like Higgs ($= \mu_H^2 H^{\dagger} H$) and Little-like Higgs coupling are found to be,

$$\mu_H^2 = \lambda^{(6)}_{\mu\mathrm{P}} \frac{f_1'^2 f_2'^2}{f'^2} \quad \text{and} \quad \lambda_{\mathrm{H}} = \lambda^{(6)}_{\mu\mathrm{P}} \frac{f_1'^4 + f_2'^4}{2 f'^4}. \tag{61}$$

The mass of SM-like Higgs as produced by radiative symmetry breaking of SU(6) via Little-like Higgses from the potential in Eq. (60), and also Eq. (61),

$$m_H^2 = \frac{g^4}{16\pi^2} \frac{\lambda^{(6)}_{\mu\mathrm{P}}}{f'^2} (f_1'^2 f_2'^2) \log \left\{ \frac{\left(\Lambda^{\mathrm{ZP}}_{(6)}\right)^2}{\mu_H^2} \right\}, \tag{62}$$

where $\Lambda^{\mathrm{ZP}}_{(6)}$ is the cut-off scale for zero-mode (perturbative) SU(6) Baby Higgs theory above which SU(6) higher-mode perturbative and non-perturbative theory govern.

From Eq.(61) with $\mu_H \sim \mathcal{O}(100\mathrm{Ge}V)$ and $f_i' \sim \mathcal{O}(1\mathrm{Te}V)$ one demonstrates that $\lambda^{(6)}_{\mu\mathrm{P}} \ll \lambda^{(6)}_{y} = g_5^2$ or $\mathcal{O}(\lambda^{(6)}_{\mu\mathrm{P}}) \sim \mathcal{O}(\lambda_H) \sim \mathcal{O}(10^{-2})$ to $\mathcal{O}(10^{-1})$ and justifies that Little-like Higgses derived from SU(6) are weakly-coupled. Therefore zero mode-perturbative quartic coupling constant is much lower compared to $\lambda^{(6)}_{y}$ with a factor $\sim \mathcal{O}(10^{-2})$.

Another SM-like Higgs emerges from Eq. (57) with the mass term $\mu^2_{H''} H''^{\dagger} H''$ which gives

$$m_{H''}^2 = \frac{g^4}{16\pi^2} \frac{\lambda^{(6)}_{\mu\mathrm{P}}}{f'^2} (f_1'^4 + f_2'^4) \log \left\{ \frac{\left(\Lambda^{\mathrm{ZP}}_{(6)}\right)^2}{\mu_{H''}^2} \right\}, \tag{63}$$

with $\mu_{H''} \sim \mathcal{O}(100\mathrm{Ge}V)$.

The factor $f_1'^2 f_2'^2$ in Eq. (62) will reach maximum value at $f_1' = f_2'$ if $f_1'^2 + f_2'^2$ is set constant and gives accordingly the interval $m_H < m_1$. On the other hand

$(f_1'^4 + f_2'^4)$ in Eq. (63) will obtain its minimum value at $f_1' = f_2'$ if $(f_1'^2 f_2'^2)$ is set constant and provides the interval $m_{H''} > m_2$. One can conclude that there exist the exclusion area for Higgs mass $m_1 < m_{\rm Higgs}({\rm excluded}) < m_2$.

Eq. (62) gives the mass of light SM-like Higgs while Eq. (63) clearly provides intermediate SM-like Higgs since $f_1'^4 + f_2'^4 > f_1'^2 f_2'^2$.

Two Little-like Higgses, H and H', of Eq. (62) and the new Higgs, H'', of Eq. (63) clearly form triplets of Higgses and when they are put in SU(6) multiplet one find two sextets $6_H = \left(H\ H'\ H''\right)^T$ and one decapentuplet $15_H = \begin{pmatrix} (0)_{3\times 3} & \begin{pmatrix} \begin{smallmatrix}0&0\\0&0\end{smallmatrix} & (H) \\ (H'^\dagger) & 0 \end{pmatrix} \\ \begin{pmatrix} \begin{smallmatrix}0&0\\0&0\end{smallmatrix} & (H') \\ (H^\dagger) & 0 \end{pmatrix} & \begin{pmatrix} \begin{smallmatrix}0&0\\0&0\end{smallmatrix} & (H'') \\ (H''^\dagger) & 0 \end{pmatrix} \end{pmatrix}$ [2,20]. With their hermitian conjugate they give totally 18 (Little-like) Higgses to be eaten by the will-be-massive gauge bosons and bring about symmetry breaking SU(6)$\to$SU(3)$\times$SU(3)$\times$U(1) .

This is basically the area of weakly-coupled SU(6) Baby Higgs i.e. $\Lambda_{(3)} < {\rm WC} < \Lambda_{(6)}^{\rm NP}$ where GUT based on SU(6) symmetry also lives and governs. (Appendix A.1)

4. Phenomenological Aspects

4.1. *Unification of gauge coupling constant*

The three couplings g_c, g_w and g_{em} in the brane $y = 0$ due to symmetry $\rm SU(3)_C\times SU(2)_L\times U(1)_Y$ run linearly in logarithmic scale even into the near-brane area below the compactification scale M_c. The reason is clear for below M_c the 4D-property is dominant ($\Lambda_{(6)}^{\rm 4D} \sim M_c$). Above M_c with more dominant 5D-property the couplings shall curve logarithmically until $M_* \sim 10^9$ TeV with value close to $0.6 - 0.7$ and continue to be power law until approaching $g_5 \sim 4\pi$ at $\Lambda_{(6)}^{\rm 5D}$.

Following ref [14] and [32] one has the following plot for $M_c \sim 10^{10}$ GeV. Now it is clear for 4D SU(6) weakly-coupled GUT based on zero mode with $\Lambda_{(6)}^{\rm ZP} \sim 1000$ TeV the coupling constants reach value $g_4 \sim 0.7$. More elaborate discussion is given in Appendix B.

4.2. *Higgs spectrum and masses*

Latest LHC data on Higgs mass excluded region lie in the interval 145-466 GeV. On the other hand the near-brane weakly-coupled SU(6) Baby Higgses do provide beyond-SM Higgses of light and heavy types. If one sets $f_1' \sim 4.0$ TeV, and $f_2' \sim 5.0$ TeV $g_4 = 0.7$ and $\mu_H \sim \mu_{H'} \sim \mu_{H''} \sim 100$ GeV and $\lambda_{\mu\rm P}^{(6)} = 0.10$ with cut-off scale $\Lambda_{(6)}^{\rm ZP} \sim 1000$ TeV then masses of Higgses are 108 GeV and 160 GeV for $H(= H')$ and H'' which are exactly (light) SM Higgs for H and excluded intermediate Higgs for H''. This proves that electroweak scale has been increased up 10 TeV. But if $f_i', i = 1, 2$ is set at $O(10$ Te$V)$ such as $f_1' = 16$ TeV and $f_2' = 20$ TeV, and other parameters remain the same then one obtains Higgs masses of 432 GeV and 640 GeV for excluded light Higgs for H and intermediate Higgs for H'' which is well under

unitary constraint of ~ 700 GeV. These provide the intermediate Higgs boson. If one adds $\delta_{m_H} \sim$ 10-20 GeV due to radiative correction the light and intermediate Higgses lie in the most preferred mass interval of Higgs boson. This result confirms that H is light with maximum $f_i' = 5.92$ TeV and H'' is intermediate (heavy) Higgs bosons with minimum $f_i' = 13.45$ TeV and proves that the excluded region corresponds to VEV interval $(5.9 - 13.45)$ TeV.

4.3. *Proton decay*

Another important phenomenological constraint is proton decay. Fortunately, proton decay in the current model can be kept long enough to fulfill the experimental bound [20]. One of the reasons the leptoquark like interaction at tree level is not allowed at all. There are actually two reasons for this behavior, i.e. at the SU(3) scale, the SU(3) triplets containing quarks and leptons generated from the SU(6) sextet are completely separated. Obviously there is no tree level interaction between both of them, and, at the SU(6) scale, as explained in [35] and shown by the subgenerators $\lambda_{C^{(1,2)}}$ in [35], all charges of leptoquark-like gauge bosons in the model are integer. This disallows tree level quark and lepton interaction which should require gauge bosons with fractional charges.

Possible decay due to baryon and lepton number violating dimension-6 operators generated by quantum correction is suppressed by $1/M_{(5\mathrm{D})}^2$ where $M_{(5\mathrm{D})} \sim \Lambda_{(6)}^{\mathrm{NP}} \sim 10^8$ GeV with $M_{(5\mathrm{D})}$ is 5D-origin GUT scale (Appendix A.2) which is equivalent to the suppression factor of dimension-5 operator in conventional 4D GUT scale $1/M_{(4\mathrm{D})}$ with $M_{(4\mathrm{D})} \sim 10^{16}$ GeV. This confirms the same proton stability as known in conventional 4D GUT. Alternative scheme for protecting proton lifetime can also be provided by this model through localizing wave function on the brane $y = 0$ for SM particles [32,36] with thickness $L = (M^*)^{-1}$ separating baryon from lepton or just putting baryon in the brane and lepton in the near-brane at the proximity beyond separating distance L. Proton decay starts to take place at M^* as low as 1.0 TeV [32,36] which is very much lower than $M^* = 10^{12}$ GeV in this model (Appendix A.2).

Therefore, roughly speaking the model should predict the proton life time close to the SM's one. Of course, more investigation should be done properly in a separate work.

5. Conclusion

The SU(6)-origin Little (Baby) Higgses as the by-product of Scherk-Schwarz mechanisms and orbifold S^1/Z_2 breaking with duality in trivial and pseudo non-trivial manners have decoupled from SU(6) GUT and been replaced immediately by SU(6) will-be-SimplestLittleHiggses and Baby Higgs. This approach is realized mainly by utilizing the zero mode terms, so that it shows up as weakly-coupled in contrast with the strongly-coupled SU(6)-origin Little (Baby) Higgses. Now, it is clear that

the dimensional deconstruction and symmetry breaking of 5D SU(6) happen almost at once due to duality condition.

This brings about the reduction (elimination) of higher modes in trivial manner and the splitting of parity in pseudo non-trivial manner. Consequently as final product, strongly-coupled SU(6)-origin Little (Baby) Higgses transform itself into final SU(3) Simplest Little Higgses which are strongly-coupled and SU(3) Little-like Higgses which are weakly coupled as will be discussed further in a separate paper [37]. These SU(6) triplet-contained Higgses, by means of radiative symmetry breaking, break SU(6) symmetry (and electroweak symmetry later on) and produce SM-like Intermediate Higgs bosons with masses ~ 470 - 650 GeVs for VEVs $14.0-20.0$ TeV and SM Light Higgs with masses $100-110$ GeV for VEVs $4.0-5.0$ TeV both at $g_4 \sim 0.7$. Unification scale is of 10^{12} GeV and compactification scale at $R^{-1} \sim 10^{10}$ GeV. Extra dimension is of the order of 10^{-24} cm. Some observables are already within reach of LHC in the current time.

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Appendix A.

Appendix A.1. *Scales for Perturbative SU(6) GUT*

Let us define Trivial Orbifold Breaking (TOB), or pseudo Non-trivial Breaking, at energy level Λ_{TOB} while (Radiative) Symmetry Breaking (SB) at energy level Λ_{SB} which is assumed to be a bit higher than Λ_3 the cut-off scale of 4D-SU(3) $\times$ SU(3) $\times$ U(1) theory. We also define Λ_{SS}, Scherk-Schwarz (SS) symmetry breaking level which is taken to be higher than Λ_{TOB}. For clarity one can draw the following energy scale and plot the valid interval for near-brane (proximity) area which represents 4D SU(6) area obtained from Sec. 2.3.1, as shown in Fig. 2.

Later on near-brane interval will be divided into perturbative SU(6) and non-perturbative SU(6) theory. We define compactification scale M_c which is slightly higher than $\Lambda^{\text{4D}}_{(6)}$, the cut-off scale of 4D-SU(6) theory, in the following equation

$$M_c = \frac{1}{R} \tag{A.1}$$

where R is the compactification radius and unification scale (of gauge coupling) M_* with the following constraint [31,32]

$$M_* R \sim \mathcal{O}(100). \tag{A.2}$$

For our purpose we may take $M_* \sim 100 M_c$ [37-38].

Following ansatz is taken: below M_c the near-brane area shows dominant 4D-property while above M_c the near-brane (now becoming bulk) has dominant 5D-property. In this way one finds perturbative approach is valid below M_c and results

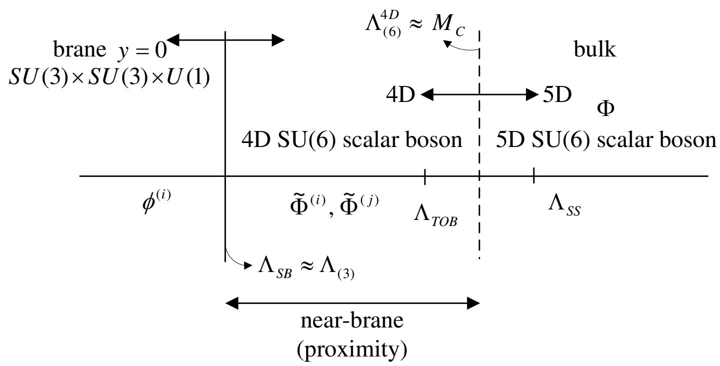

Fig. 2. Near-brane 4D-SU(6) scalar boson cut-off scale.

in so-called weakly-coupled SU(6) Baby Higgs which is suitable for SU(6) GUT from Λ_3 up to $\Lambda^{\rm NP}_{(6)} \sim M_{(5{\rm D})}$, the cut-off scale of 5D-origin SU(6) GUT theory. On the other hand non-perturbative approach must be taken above $\Lambda^{\rm NP}_{(6)}$ due to more (and more) Kaluza-Klein states causing more dominant 5D-property. Higher modes at maximum will bring about the strongly-coupled property of the SU(6)-origin Little (Baby) Higgs, a by-product of this model, serving as the origin of SU(6) Baby Higgs. It is clear later on that the interval $(\Lambda_3, \Lambda^{\rm NP}_{(6)})$ does consist of $\Lambda^{\rm ZP}_{(6)}, \Lambda^{\rm FP}_{(6)}, \cdots$ etc, as the cut-off scale of zero mode-based SU(6) GUT and first mode-based SU(6) GUT, etc., as shown in Fig. 3.

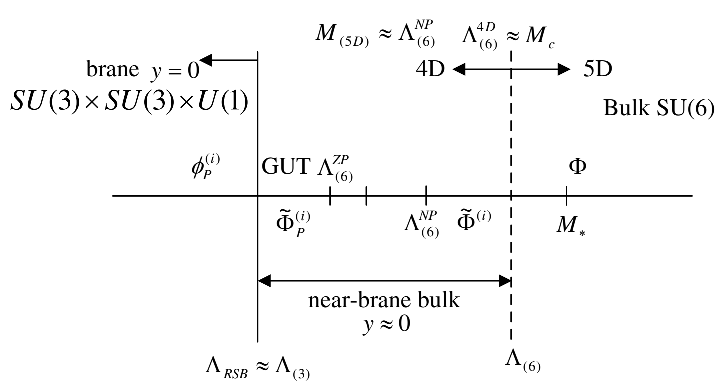

Fig. 3. SU(6) weakly-coupled GUT and SU(6)-origin strongly-coupled Little (Baby) Higgs.

Finally the zero mode-based weakly-coupled SU(6) Baby Higgses are given in the Fig. 4 below.

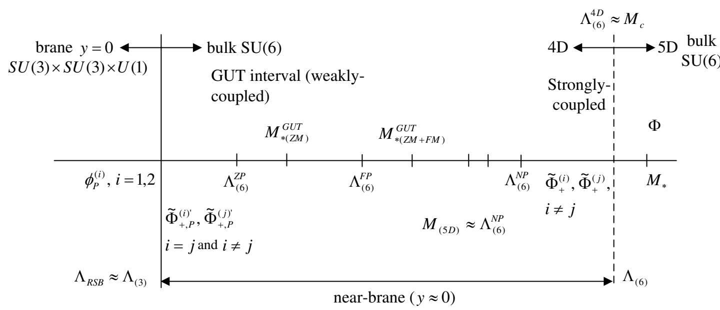

Fig. 4. Zero mode- and first mode- based weakly-coupled SU(6) Baby Higgs in the near-brane ($y \sim 0$).

Appendix A.2. *Order of Estimate*

Planck scale is reduced significantly in 5D and so is GUT scale following the formula below,

$$\left[M_P^{(5D)}\right]^{D-2} = \left[M_P^{(4D)}\right]^2 / R^\delta \tag{A.3}$$

where

$D = 4 + \delta = 5$, is space-time dimension;
$\delta \ = 1$, is number of extra-dimension;
$\frac{1}{R} = M_c$ is compactification scale.

With $M_P^{(4D)} \sim 10^{19}$ GeV and M_c is assumed to be 10^{10} GeV one finds the five-dimensional Planck scale, $(M_P^{(5D)})$ in the order of $\sim 10^{16}$ GeV or $10^{-3} M_P^{(4D)}$. This brings us to the 5D-SU(6)GUT unification scale of 10^{12} Ge$V(M_*)$ and compactification scale $M_c \sim 10^{-2} M_* \sim 10^{10}$ GeV which has justified the previous assumption. This in turn establishes the limit for the SU(6) GUT perturbative cut-off scale, $M_{(\mathrm{5D})} = 10^8\mathrm{Ge}V \sim \Lambda^{\mathrm{NP}}_{(6)} < \Lambda^{\mathrm{4D}}_{(6)} \sim 10^7$ TeV and minimal cut-off scale of 5D SU(6)theory, Λ_6^{5D}, as 3×10^{10} TeV due to $(\Lambda_6/M_*)|_{\delta=1} < 30$ [37,38].

Appendix B. Unification of Gauge Coupling Constant

In general the logarithmic form of running coupling constant in 4D is changed into a power law due to the effect of extra dimension in accordance with the following formula [37],

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(\mu) - \frac{b_i - \widetilde{b}_i}{2\pi} \ln \frac{\Lambda}{\mu} - \frac{\widetilde{b}_i}{4\pi} \int_{r\Lambda^{-2}}^{r\mu^{-2}} \frac{dt}{t} \left\{ v_3 \left(\frac{it}{\pi R^2} \right) \right\}^\delta \tag{B.1}$$

where Jacobi theta-function $v_3(\tau) \equiv \sum_{n=-\infty}^{+\infty} \exp(\pi i \tau n^2)$ reflects the sum over K-K states, $\widetilde{b}_i$ are the beta-function coefficients, $r \equiv \pi (X_\delta)^{-2/\delta}$ with $X_\delta = 2\pi^{\delta/2}/\delta\Gamma(\delta/2)$ as overall normalization.

This can be modified into the following

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(R^{-1}) - \frac{b_i - \widetilde{b}_i}{2\pi} \ln(\Lambda R) - \frac{\widetilde{b}_i X_\delta}{2\pi\delta} \Big[(\Lambda R)^\delta - 1 \Big] \tag{B.2}$$

where the cut-off scale $\Lambda \gg \frac{1}{R}$ or SU(6) cut off scale $\Lambda^{5D}_{(6)}$ is set in such a way $\Lambda^{5D}_{(6)} \gg M_c$. Above M_c the last term of (B.2) becomes more and more influential. This gives rise to power-law evolution which is basically valid until the limit $\Lambda R \approx 1$ or $\Lambda \approx R^{-1} = M_c$. Here we denote Λ as $\Lambda^{5D}_{(6)}$ which is the cut-off scale of 5D-SU(6) starting from $\Lambda^{5D}_{(6)}$ downward to where $\Lambda^{4D}_{(6)} \approx M_c$. Below M_c in GUT area, perturbative approach becomes reliable which basically provides the weakly-coupled area down to symmetry breaking level $\Lambda_{(3)}$. Result is plotted, based on [14] and [38] as in in Fig. 5.

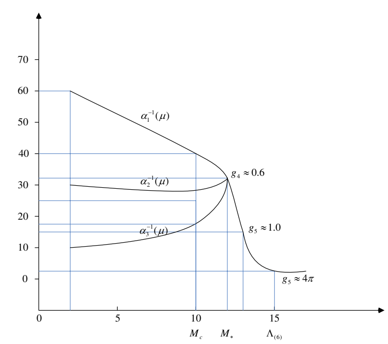

Fig. 5. Unification of gauge couplings with a single extra dimension of radius $R^{-1} \approx 10^{10}$ GeV.

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